# $T$-odd correlations in the decay of scalar fermions 

A. Bartl ${ }^{1}$, H. Fraas ${ }^{2}$, T. Kernreiter ${ }^{1,3}$, O. Kittel $^{2}$<br>${ }^{1}$ Institut für Theoretische Physik, Univ. Wien, 1090 Vienna, Austria<br>${ }^{2}$ Institut für Theoretische Physik und Astrophysik, Univ. Würzburg, 97074 Würzburg, Germany<br>${ }^{3}$ Instituto de Física Corpuscular-C.S.I.C., Univ. de València, València 46100, Spain

Received: 2 July 2003 / Revised version: 7 November 2003 /
Published online: 13 February 2004 - © Springer-Verlag / Società Italiana di Fisica 2004


#### Abstract

We define a $C P$-sensitive asymmetry in the sfermion decays $\tilde{f} \rightarrow f \tilde{\chi}_{j}^{0} \ell \bar{\ell}, f \tilde{\chi}_{j}^{0} q \bar{q}$, based on triple product correlations between the momenta of the outgoing fermions. We study this asymmetry in the MSSM with complex parameters. We show that the asymmetry is sensitive to the phases of the parameters $\mu$ and $M_{1}$. The leading contribution stems from the decay chain $\tilde{f} \rightarrow f \tilde{\chi}_{j}^{0} \rightarrow f \tilde{\chi}_{1}^{0} Z \rightarrow f \tilde{\chi}_{1}^{0} \ell \bar{\ell}\left(f \tilde{\chi}_{1}^{0} q \bar{q}\right)$, for which we obtain analytic formulae for the amplitude squared. The asymmetry can go up to $3 \%$ for $\tilde{f} \rightarrow f \tilde{\chi}_{1}^{0} \ell \bar{\ell}$, and up to $20 \%$ for $\tilde{f} \rightarrow f \tilde{\chi}_{1}^{0} q \bar{q}$. We also estimate the rates necessary to measure the asymmetry.


## 1 Introduction

In the standard model (SM) the source of $C P$-violation is given by the phase in the Kobayashi-Maskawa matrix [1]. However, it has been argued that this source is not enough to explain the observed baryon asymmetry of the universe (see for example [2]) and new sources of $C P$-violation have to be introduced. In the minimal supersymmetric extension of the SM (MSSM), several supersymmetric (SUSY) breaking parameters and the higgsino mass parameter can be complex.

The phases of the SUSY parameters are restricted by the experimental upper limits on the electric dipole moments (EDMs) [3] of electron, neutron and the ${ }^{199} \mathrm{Hg}$ and ${ }^{205} \mathrm{Tl}$ atoms. The limiting bounds are $\left|d_{e}\right|<4.3 \times 10^{-27} e \mathrm{~cm}$ $[4],\left|d_{n}\right|<6.3 \times 10^{-26} e \mathrm{~cm}[5],\left|d_{\mathrm{Hg}}\right|<2.1 \times 10^{-28} e \mathrm{~cm}[6]$ and $\left|d_{\mathrm{Tl}}\right|<1.3 \times 10^{-24} \mathrm{ecm}[7]$, respectively. The general consensus is that one of the following three conditions has to be realized:
(i) the phases are severely suppressed $[8,9]$;
(ii) supersymmetric particles of the first two generations are rather heavy, with masses of the order of a TeV [10];
(iii) there are strong cancellations between the different SUSY contributions to the EDMs [11].

At one-loop level, for the electron EDM these are the neutralino and chargino contributions, and for the neutron EDM in addition also the gluino exchange contributes. While the phase of $\mu$ is restricted, $\left|\varphi_{\mu}\right| \lesssim \pi / 10$, there is no such restriction on the phase of $M_{1}$ [12].

In order to clarify the situation an unambiguous determination of the SUSY phases is necessary. In particular, for determining also the sign of the phases, measurements of $C P$-sensitive observables are necessary. The SUSY phases give rise to $C P$-odd ( $T$-odd) observables already at tree
level [13-16]. An important class of such observables involves triple product correlations [17,18]. They allow us to define various $C P$-asymmetries which are sensitive to the different $C P$-phases. These observables could be measured at future linear collider experiments [19] and would allow us to independently determine the values of the phases.

In this paper we consider a $T$-odd correlation in the decays

$$
\begin{equation*}
\tilde{f} \rightarrow f \tilde{\chi}_{j}^{0} \ell \bar{\ell}, f \tilde{\chi}_{j}^{0} q \bar{q} \tag{1}
\end{equation*}
$$

with $\ell=e, \mu, \tau$, and $q$ denotes a quark. The $T$-odd correlation for the leptonic decay is defined as

$$
\begin{equation*}
O_{\mathrm{odd}}^{\ell}=\mathbf{p}_{f} \cdot\left(\mathbf{p}_{\ell} \times \mathbf{p}_{\bar{\ell}}\right) \tag{2}
\end{equation*}
$$

and that for the hadronic decay as

$$
\begin{equation*}
O_{\mathrm{odd}}^{q}=\mathbf{p}_{f} \cdot\left(\mathbf{p}_{q} \times \mathbf{p}_{\bar{q}}\right) \tag{3}
\end{equation*}
$$

where $\mathbf{p}$ denotes the three-momentum of the corresponding fermion. We define the corresponding $T$-odd asymmetries as

$$
\begin{align*}
\mathcal{A}_{T}^{\ell, q} & =\frac{\Gamma\left(O_{\mathrm{odd}}^{\ell, q}>0\right)-\Gamma\left(O_{\mathrm{odd}}^{\ell, q}<0\right)}{\Gamma\left(O_{\mathrm{odd}}^{\ell, q}>0\right)+\Gamma\left(O_{\mathrm{odd}}^{\ell, q}<0\right)} \\
& =\frac{\int \operatorname{sgn}\left[O_{\mathrm{odd}}^{\ell, q}\right]\left|\mathcal{M}^{\ell, q}\right|^{2} \mathrm{dLips}}{\int\left|\mathcal{M}^{\ell, q}\right|^{2} \mathrm{dLips}} \tag{4}
\end{align*}
$$

where $\mathcal{M}^{\ell, q}$ is the matrix element for the decay (1). For the measurement of $\mathcal{A}_{T}^{\ell}$ or $\mathcal{A}_{T}^{q}$ it is necessary to be able to distinguish between the charges of $\ell^{+}$and $\ell^{-}$or $q$ and $\bar{q}$. In the case $\ell=e, \mu, \tau$ this should be possible experimentally on an event by event basis at an $e^{+} e^{-}$linear collider [19]. $\mathcal{A}_{T}^{q}$ may be measurable in the case of $q=c, b$, where flavor


Fig. 1. Schematic picture of the subsequent two-body decays $\tilde{f} \rightarrow f \tilde{\chi}_{j}^{0}, \quad \tilde{\chi}_{j}^{0} \rightarrow Z \tilde{\chi}_{1}^{0}, Z \rightarrow \ell \bar{\ell}(q \bar{q})$ in the $\tilde{f}$ rest frame
reconstruction is possible [20]. However, this will only be possible statistically for a given event sample.

The leading contribution to the triple products (2) and (3) originates from the decay chain

$$
\begin{equation*}
\tilde{f} \rightarrow f \tilde{\chi}_{j}^{0} \rightarrow f \tilde{\chi}_{1}^{0} Z \rightarrow f \tilde{\chi}_{1}^{0} \ell \bar{\ell} \quad\left(f \tilde{\chi}_{1}^{0} q \bar{q}\right) \tag{5}
\end{equation*}
$$

which is shown schematically in Fig. 1. Essentially, the triple products (2) and (3) are correlations between the $\tilde{\chi}_{j}^{0}$ polarization and the $Z$ boson polarization, which are encoded in the momentum vectors of the final leptons or quarks. The correlations would vanish, for example, if a scalar particle in place of the $Z$ boson is exchanged. Final state interactions may also contribute to $\mathcal{A}_{T}^{\ell, q}$; however, they arise only at one-loop level. We expect that such contributions are smaller than $10 \%$, because only weak corrections to the absorptive part of the $\tilde{\chi}_{j}^{0}-Z-\tilde{\chi}_{1}^{0}$ vertex have to be included. In the similar case of the decay $\tilde{\chi}_{j}^{ \pm} \rightarrow W^{ \pm} \tilde{\chi}_{1}^{0}$, corrections smaller than $10 \%$ have been obtained [21]. Corrections of this order of magnitude have also been found in [22], where next-to-leading order effects on polarization observables within the SM have been studied.

As will be shown, the tree-level contribution to the triple product correlations (2) and (3) are proportional to the imaginary part of the $\tilde{\chi}_{j}^{0}-Z-\tilde{\chi}_{1}^{0}$ coupling squared and are sensitive to the phases of the neutralino mass parameters $M_{1}$ and $\mu$ (see (25)-(34)). $O_{\text {odd }}^{\ell, q}$ is not sensitive to the trilinear scalar coupling parameter $A_{f}$ of the sfermion $\tilde{f}$. The reason is that the first decay in the chain (5), $\tilde{f} \rightarrow f \chi_{j}^{0}$, is a two-body decay of a scalar particle. In order to be sensitive to the phase of $A_{f}$ one would have to construct instead of (2) and (3) a triple product correlation involving the transverse polarization of the fermion $f$. In principle this could be possible for $f=\tau, t$ [23], but this case will not be considered here (for a similar case see [15]). The $T$-odd correlation (2) was proposed in [16] and the size of the asymmetry was calculated for the decay $\tilde{\mu} \rightarrow \mu \tilde{\chi}_{2}^{0} \rightarrow$ $\chi_{1}^{0} \mu \ell^{+} \ell^{-}$; however, for a specific final state configuration only. In the present paper we extend the work of [16] by calculating the asymmetries (4) in the whole phase space.

In Sect. 2 we give the definitions and the matrix element of the decay we are interested in. In Sect. 3 we perform the calculation of the $T$-odd asymmetry. In Sect. 4 we present our numerical results. Section 5 contains a short summary and conclusion.

## 2 Definitions and formalism

### 2.1 Lagrangian and couplings

The parts of the interaction Lagrangian of the MSSM relevant for decay (5) are (in our notation and conventions we follow closely $[24,25]$ )

$$
\begin{gather*}
\mathcal{L}_{\tilde{f} f \tilde{\chi}_{j}^{0}}=\tilde{f}_{k} \bar{f}\left(b_{k j}^{\tilde{f}} P_{L}+a_{k j}^{\tilde{f}} P_{R}\right) \tilde{\chi}_{j}^{0}+\text { h.c. } \\
j=1, \ldots, 4, k=1,2 \tag{6}
\end{gather*}
$$

where

$$
a_{k j}^{\tilde{f}}=g\left(\mathcal{R}_{k n}^{\tilde{f}}\right)^{*} \mathcal{A}_{j n}^{f}, \quad b_{k j}^{\tilde{f}}=g\left(\mathcal{R}_{k n}^{\tilde{f}}\right)^{*} \mathcal{B}_{j n}^{f}, \quad(n=L, R)
$$

$$
\begin{equation*}
\mathcal{A}_{j}^{\ell, q}=\binom{f_{L j}^{\ell, q}}{h_{R j}^{\ell, q}}, \quad \mathcal{B}_{j}^{\ell, q}=\binom{h_{L j}^{\ell, q}}{f_{R j}^{\ell, q}} \tag{7}
\end{equation*}
$$

with

$$
\begin{align*}
h_{L j}^{\ell} & =\left(h_{R j}^{\ell}\right)^{*}=Y_{\ell} N_{j 3}^{*} \\
f_{L j}^{\ell} & =-\frac{1}{\sqrt{2}}\left(\tan \Theta_{\mathrm{W}} N_{j 1}+N_{j 2}\right), \\
f_{R j}^{\ell} & =\sqrt{2} \tan \Theta_{\mathrm{W}} N_{j 1}^{*}  \tag{9}\\
h_{L j}^{u} & =\left(h_{R j}^{u}\right)^{*}=Y_{u} N_{j 4}^{*} \\
f_{L j}^{u} & =\frac{1}{\sqrt{2}}\left(\tan \Theta_{\mathrm{W}} N_{j 2}+N_{j 1}\right), \\
f_{R j}^{u} & =-\frac{2 \sqrt{2}}{3} \tan \Theta_{\mathrm{W}} N_{j 1}^{*}  \tag{10}\\
h_{L j}^{d} & =\left(h_{R j}^{d}\right)^{*}=Y_{d} N_{j 3}^{*} \\
f_{L j}^{d} & =-\frac{1}{\sqrt{2}}\left(\frac{1}{3} \tan \Theta_{\mathrm{W}} N_{j 2}-N_{j 2}\right) \\
f_{R j}^{d} & =\frac{\sqrt{2}}{3} \tan \Theta_{\mathrm{W}} N_{j 1}^{*} \tag{11}
\end{align*}
$$

and

$$
\begin{equation*}
Y_{\ell, d}=\frac{m_{\ell, d}}{\sqrt{2} m_{W} \cos \beta}, \quad Y_{u}=\frac{m_{u}}{\sqrt{2} m_{W} \sin \beta} . \tag{12}
\end{equation*}
$$

Here, $P_{L, R}=1 / 2\left(1 \mp \gamma_{5}\right)$, $g$ denotes the weak coupling constant, $\Theta_{\mathrm{W}}$ is the weak mixing angle, $m_{W}$ is the mass of the $W$ boson and $\mathcal{R}_{k n}^{\tilde{f}}$ is the scalar fermion mixing matrix defined below. $m_{\ell}$ and $m_{u}\left(m_{d}\right)$ is the mass of the corresponding lepton and up-type (down-type) quark, respectively. $N_{i j}$ is the complex unitary $4 \times 4$ matrix which diagonalizes the neutral gaugino-higgsino mass matrix $Y_{\alpha \beta}$, $N_{i \alpha}^{*} Y_{\alpha \beta} N_{k \beta}^{*}=m_{\tilde{\chi}_{i}^{0}} \delta_{i k}$, in the basis $\left(\tilde{B}, \tilde{W}^{3}, \tilde{H}_{1}^{0}, \tilde{H}_{2}^{0}\right)[24]$. The masses and couplings of the $\tilde{f}_{k}$ follow from the hermitian sfermion mass matrix which in the basis $\left(\tilde{f}_{L}, \tilde{f}_{R}\right)$ reads

$$
\mathcal{L}_{M}^{\tilde{f}}=-\left(\tilde{f}_{L}^{\dagger}, \tilde{f}_{R}^{\dagger}\right)\left(\begin{array}{cc}
M_{\tilde{f}_{L L}}^{2} & \mathrm{e}^{-\mathrm{i} \varphi_{\tilde{f}}}\left|M_{\tilde{f}_{L R}}^{2}\right|  \tag{13}\\
\mathrm{e}^{\mathrm{i} \varphi_{\tilde{f}} \mid}\left|M_{\tilde{f}_{L R}}^{2}\right| & M_{\tilde{f}_{R R}}^{2}
\end{array}\right)\binom{\tilde{f}_{L}}{\tilde{f}_{R}},
$$

with

$$
\begin{align*}
M_{\tilde{f}_{L L}}^{2} & =M_{L \tilde{f}}^{2}+\left(I_{3 L}^{f}-q_{f} \sin ^{2} \Theta_{\mathrm{W}}\right) \cos 2 \beta m_{Z}^{2}+m_{f}^{2}  \tag{14}\\
M_{\tilde{f}_{R R}}^{2} & =M_{R \tilde{f}}^{2}+q_{f} \sin ^{2} \Theta_{\mathrm{W}} \cos 2 \beta m_{Z}^{2}+m_{f}^{2},  \tag{15}\\
M_{\tilde{f}_{R L}}^{2} & =\left(M_{\tilde{f}_{L R}}^{2}\right)^{*}=m_{f}\left(A_{f}-\mu^{*}(\cot \beta)^{2 I_{3 L}^{f}}\right),  \tag{16}\\
\varphi_{\tilde{f}} & =\arg \left[A_{f}-\mu^{*}(\cot \beta)^{2 I_{3 L}^{f}}\right], \tag{17}
\end{align*}
$$

where $m_{Z}$ is the mass of the $Z$ boson, $q_{f}$ and $I_{3 L, R}^{f}$ is the charge and the isospin of the fermion, respectively. $M_{L \tilde{f}}, M_{R \tilde{f}}, A_{f}$ are the soft SUSY-breaking parameters of the $\tilde{f}_{i}$ system. The $\tilde{f}$ mass eigenstates are $\left(\tilde{f}_{1}, \tilde{f}_{2}\right)=$ $\left(\tilde{f}_{L}, \tilde{f}_{R}\right) \mathcal{R}^{\tilde{f}^{T}}$ with

$$
\mathcal{R}^{\tilde{\tau}}=\left(\begin{array}{cc}
\mathrm{e}^{\mathrm{i} \varphi_{\tilde{f}}} \cos \theta_{\tilde{f}} & \sin \theta_{\tilde{f}}  \tag{18}\\
-\sin \theta_{\tilde{f}} & \mathrm{e}^{-\mathrm{i} \varphi_{\tilde{f}}} \cos \theta_{\tilde{f}}
\end{array}\right)
$$

and

$$
\begin{align*}
\cos \theta_{\tilde{f}} & =\frac{-\left|M_{\tilde{f}_{L R}}^{2}\right|}{\sqrt{\left|M_{\tilde{f}_{L R}}^{2}\right|^{2}+\left(m_{\tilde{f}_{1}}^{2}-M_{\tilde{f}_{L L}}^{2}\right)^{2}}} \\
\sin \theta_{\tilde{f}} & =\frac{M_{\tilde{f}_{L L}}^{2}-m_{\tilde{f}_{1}}^{2}}{\sqrt{\left|M_{\tilde{f}_{L R}}^{2}\right|^{2}+\left(m_{\tilde{f}_{1}}^{2}-M_{\tilde{f}_{L L}}^{2}\right)^{2}}} \tag{19}
\end{align*}
$$

The mass eigenvalues are

$$
\begin{aligned}
& m_{\tilde{f}_{1,2}}^{2} \\
& =\frac{1}{2}\left(\left(M_{\tilde{f}_{L L}}^{2}+M_{\tilde{f}_{R R}}^{2}\right) \mp \sqrt{\left(M_{\tilde{f}_{L L}}^{2}-M_{\tilde{f}_{R R}}^{2}\right)^{2}+4\left|M_{\tilde{f}_{L R}}^{2}\right|^{2}}\right)
\end{aligned}
$$

The remaining parts of the interaction Lagrangian of the MSSM relevant for the decay (5) are

$$
\begin{gather*}
\mathcal{L}_{Z \tilde{\chi}_{i}^{0} \tilde{\chi}_{j}^{0}}=Z^{\mu} \bar{\chi}_{i}^{0} \gamma_{\mu}\left(O_{i j}^{\prime \prime L} P_{L}+O_{i j}^{\prime \prime R} P_{R}\right) \tilde{\chi}_{j}^{0}  \tag{21}\\
i, j=1, \ldots, 4
\end{gather*}
$$

and

$$
\begin{equation*}
\mathcal{L}_{Z f \bar{f}}=Z^{\mu} \bar{f} \gamma_{\mu}\left(L_{f} P_{L}+R_{f} P_{R}\right) f \tag{22}
\end{equation*}
$$

respectively, where

$$
\begin{align*}
O_{i j}^{\prime \prime L} & =\frac{g}{4 \cos \Theta_{\mathrm{W}}}\left(N_{i 4} N_{j 4}^{*}-N_{i 3} N_{j 3}^{*}\right), \\
O_{i j}^{\prime \prime R} & =-O_{i j}^{\prime \prime L^{*}}  \tag{23}\\
L_{f}\left(R_{f}\right) & =-\frac{g}{\cos \Theta_{\mathrm{W}}}\left(I_{3 L(R)}^{f}-q_{f} \sin ^{2} \Theta_{\mathrm{W}}\right) . \tag{24}
\end{align*}
$$

Note that our definition of $O_{i j}^{\prime \prime L, R}\left(L_{f}, R_{f}\right)$ differs from that given in [24] by a factor $g / 2 \cos \Theta_{\mathrm{W}}\left(g / \cos \Theta_{\mathrm{W}}\right)$.

### 2.2 Spin-density matrix formalism

For the calculation of the amplitude squared of the subsequent two-body decays (5) of the sfermion, we use the spindensity matrix formalism $[26,27]$. The amplitude squared is given by

$$
\begin{align*}
& |\mathcal{M}|^{2}=\left|\Delta\left(\tilde{\chi}_{j}^{0}\right)\right|^{2}|\Delta(Z)|^{2} \\
& \times \sum_{\lambda_{i}, \lambda_{i}^{\prime}, \lambda_{k}, \lambda_{k}^{\prime}}\left(\rho_{D_{1}}\right)_{\lambda_{i} \lambda_{i}^{\prime}}\left(\rho_{D_{2}}\right)_{\lambda_{k} \lambda_{k}^{\prime}}^{\lambda_{i}^{\prime} \lambda_{i}}\left(\rho_{D_{3}}\right)^{\lambda_{k}^{\prime} \lambda_{k}} \tag{25}
\end{align*}
$$

with the propagators $\Delta\left(\tilde{\chi}_{j}^{0}\right)=1 /\left[p_{\chi_{j}}^{2}-m_{\chi_{j}}^{2}+\mathrm{i} m_{\chi_{j}} \Gamma_{\chi_{j}}\right]$ and $\Delta(Z)=1 /\left[p_{Z}^{2}-m_{Z}^{2}+\mathrm{i} m_{Z} \Gamma_{Z}\right]$. Here, $p_{\chi_{j}}, m_{\chi_{j}}, \Gamma_{\chi_{j}}$ $\left(p_{Z}, m_{Z}, \Gamma_{Z}\right)$ are the four-momenta, masses and widths of the decaying neutralino ( $Z$ boson), respectively. The amplitude squared is composed of the unnormalized spindensity matrices $\rho_{D_{1}}, \rho_{D_{2}}$ and $\rho_{D_{3}}$ of the decay (5), which carry the helicity indices $\lambda_{i}, \lambda_{i}^{\prime}$ of the neutralinos and/or the helicity indices $\lambda_{k}, \lambda_{k}^{\prime}$ of the $Z$ boson. Introducing a set of polarization basis four-vectors $s_{\chi_{j}}^{a}(a=1,2,3)$ for the neutralino $\tilde{\chi}_{j}^{0}$, which fulfill the orthonormality relations $s_{\chi_{j}}^{a} \cdot s_{\chi_{j}}^{b}=-\delta^{a b}$ and $s_{\chi_{j}}^{a} \cdot p_{\chi_{j}}=0$, the density matrices can be expanded in terms of the Pauli matrices:

$$
\begin{align*}
\left(\rho_{D_{1}}\right)_{\lambda_{i} \lambda_{i}^{\prime}} & =\delta_{\lambda_{i} \lambda_{i}^{\prime}} D_{1}+\sigma_{\lambda_{i} \lambda_{i}^{\prime}}^{a} \Sigma_{D_{1}}^{a},  \tag{26}\\
\left(\rho_{D_{2}}\right)_{\lambda_{k}, \lambda_{k}^{\prime}}^{\lambda_{i}^{\prime}, \lambda_{i}} & =\left[\delta_{\lambda_{i}^{\prime} \lambda_{i}} D_{2}^{\mu \nu}+\sigma_{\lambda_{i}^{\prime} \lambda_{i}}^{b} \Sigma_{D_{2}}^{b \mu \nu}\right] \varepsilon_{\mu}^{\lambda_{k} *} \varepsilon_{\nu}^{\lambda_{k}^{\prime}},  \tag{27}\\
\left(\rho_{D_{3}}\right)^{\lambda_{k}^{\prime} \lambda_{k}} & =D_{3}^{\rho \sigma} \varepsilon_{\sigma}^{\lambda_{k}^{\prime} *} \varepsilon_{\rho}^{\lambda_{k}} . \tag{28}
\end{align*}
$$

The polarization vectors $\varepsilon_{\mu}^{\lambda_{k}}$ of the $Z$ boson obey $p_{Z}^{\mu} \varepsilon_{\mu}^{\lambda_{k}}=0$ and the completeness relation $\sum_{\lambda_{k}} \varepsilon_{\mu}^{\lambda_{k} *} \varepsilon_{\nu}^{\lambda_{k}}=-g_{\mu \nu}+$ $p_{Z \mu} p_{Z \nu} / m_{Z}^{2}$. The expansion coefficients of the density matrices (26)- (28) are

$$
\begin{align*}
D_{1}= & \left(\left|a_{k j}^{\tilde{f}}\right|^{2}+\left|b_{k j}^{\tilde{f}}\right|^{2}\right)\left(p_{f^{\prime}} \cdot p_{\chi_{j}}\right),  \tag{29}\\
\Sigma_{D_{1}}^{a}= & m_{\chi_{j}}\left(\left|b_{k j}^{\tilde{f}}\right|^{2}-\left|a_{k j}^{\tilde{f}}\right|^{2}\right)\left(p_{f^{\prime}} \cdot s_{\chi_{j}}^{a}\right),  \tag{30}\\
D_{2 \rho \sigma}= & 4 g_{\rho \sigma}\left[2 \operatorname{Re}\left(O_{1 j}^{\prime \prime L} O_{1 j}^{\prime \prime R^{*}}\right) m_{\chi_{1}} m_{\chi_{j}}\right. \\
& \left.\quad-\left(\left|O_{1 j}^{\prime \prime L}\right|^{2}+\left|O_{1 j}^{\prime \prime R}\right|^{2}\right)\left(p_{\chi_{1}} \cdot p_{\chi_{j}}\right)\right]  \tag{31}\\
+ & 4\left(\left|O_{1 j}^{\prime \prime L}\right|^{2}+\left|O_{1 j}^{\prime \prime R}\right|^{2}\right)\left(p_{\chi_{j} \rho} p_{\chi_{1} \sigma}+p_{\chi_{j} \sigma} p_{\chi_{1} \rho}\right), \\
\Sigma_{D_{2} \rho \sigma}^{a}= & \mathrm{i} 8 m_{\chi_{1}} \operatorname{Im}\left(O_{1 j}^{\prime \prime L} O_{1 j}^{\prime \prime R^{*}}\right)\left(p_{\chi_{j} \rho} s_{\chi_{j} \sigma}^{a}-p_{\chi_{j} \sigma} s_{\chi_{j} \rho}^{a}\right) \\
+ & \mathrm{i} 4 \varepsilon_{\rho \sigma \mu \nu} p_{\chi_{1}}^{\mu} s_{\chi_{j}}^{a \nu} m_{\chi_{j}}\left(\left|O_{1 j}^{\prime \prime L}\right|^{2}+\left|O_{1 j}^{\prime \prime R}\right|^{2}\right) \\
- & \mathrm{i} 8 \varepsilon_{\rho \sigma \mu \nu} p_{\chi_{j}}^{\mu} s_{\chi_{j}}^{a \nu} m_{\chi_{1}} \operatorname{Re}\left(O_{1 j}^{\prime \prime L} O_{1 j}^{\prime \prime R^{*}}\right),  \tag{32}\\
D_{3}^{\rho \sigma}= & -2 g^{\rho \sigma}\left(L_{f}^{2}+R_{f}^{2}\right)\left(p_{f} \cdot p_{\bar{f}}\right) \\
+ & 2\left(p_{f}^{\rho} p_{\bar{f}}^{\sigma}+p_{\bar{f}}^{\rho} p_{f}^{\sigma}\right)\left(L_{f}^{2}+R_{f}^{2}\right)
\end{align*}
$$

$$
\begin{equation*}
+\mathrm{i} 2\left(R_{f}^{2}-L_{f}^{2}\right) \varepsilon^{\rho \sigma \mu \nu} p_{f \mu} p_{\bar{f} \nu} \tag{33}
\end{equation*}
$$

with $\varepsilon^{0123}=1$ and $m_{\chi_{1}}$ the mass of the lightest supersymmetric particle (LSP). The masses of the fermions $f=e, \mu, \tau, c, b$ are set to zero. In (29) and (30) $f^{\prime}$ denotes the fermion stemming from the first decay in (5). Inserting the density matrices (26)- (28) in (25), the amplitude squared is given by
$|\mathcal{M}|^{2}=2\left|\Delta\left(\tilde{\chi}_{j}^{0}\right)\right|^{2}|\Delta(Z)|^{2}\left\{D_{1} D_{2 \rho \sigma}+\Sigma_{D_{1}}^{a} \Sigma_{D_{2} \rho \sigma}^{a}\right\} D_{3}^{\rho \sigma}$.

## 3 T-odd asymmetry

In the following we present in some detail the calculation of the $T$-odd asymmetry in (4) for the slepton decays $\tilde{\ell} \rightarrow$ $\ell \tilde{\chi}_{j}^{0} \rightarrow \ell \tilde{\chi}_{1}^{0} Z \rightarrow \ell \tilde{\chi}_{1}^{0} f \bar{f}$. The replacements which must be made for the asymmetry in the case of $\tilde{q}$ decays are obvious.

In the rest frame of $\tilde{\ell}$, the coordinate system is defined such that the momentum four-vectors are given by

$$
\begin{align*}
p_{Z} & =\left(E_{Z}, 0,0,\left|\mathbf{p}_{Z}\right|\right) \\
p_{\chi_{j}} & =\left|\mathbf{p}_{\chi_{j}}\right|\left(E_{\chi_{j}} /\left|\mathbf{p}_{\chi_{j}}\right|, \sin \theta_{1}, 0, \cos \theta_{1}\right)  \tag{35}\\
p_{\bar{f}} & =\left|\mathbf{p}_{\bar{f}}\right|\left(E_{\bar{f}} /\left|\mathbf{p}_{\bar{f}}\right|, \sin \theta_{2} \cos \phi_{2}, \sin \theta_{2} \sin \phi_{2}, \cos \theta_{2}\right)
\end{align*}
$$

where

$$
\begin{equation*}
\left|\mathbf{p}_{\chi_{j}}\right|=\frac{m_{\tilde{\ell}}^{2}-m_{\chi_{j}}^{2}}{2 m_{\tilde{\ell}}}, \quad\left|\mathbf{p}_{\bar{f}}\right|=\frac{m_{Z}^{2}}{2\left(E_{Z}-\left|\mathbf{p}_{Z}\right| \cos \theta_{2}\right)} \tag{36}
\end{equation*}
$$

and

$$
\begin{align*}
& \left|\mathbf{p}_{Z}^{ \pm}\right|=\left[\left(m_{\chi_{j}}^{2}+m_{Z}^{2}-m_{\chi_{1}}^{2}\right)\left|\mathbf{p}_{\chi_{j}}\right| \cos \theta_{1}\right. \\
& \left. \pm E_{\chi_{j}} \sqrt{\lambda\left(m_{\chi_{j}}^{2}, m_{Z}^{2}, m_{\chi_{1}}^{2}\right)-4\left|\mathbf{p}_{\chi_{j}}\right|^{2} m_{Z}^{2}\left(1-\cos ^{2} \theta_{1}\right)}\right] \\
& \quad /\left[2\left|\mathbf{p}_{\chi_{j}}\right|^{2}\left(1-\cos ^{2} \theta_{1}\right)+2 m_{\chi_{j}}^{2}\right] \tag{37}
\end{align*}
$$

with $\lambda(x, y, z)=x^{2}+y^{2}+z^{2}-2(x y+x z+y z)$. There are two solutions $\left|\mathbf{p}_{Z}^{ \pm}\right|$in the case $\left|\mathbf{p}_{\chi_{j}}^{0}\right|<\left|\mathbf{p}_{\chi_{j}}\right|$, where $\left|\mathbf{p}_{\chi_{j}}^{0}\right|=$ $\sqrt{\lambda\left(m_{\chi_{j}}^{2}, m_{Z}^{2}, m_{\chi_{1}}^{2}\right)} / 2 m_{Z}$ is the neutralino momentum if the $Z$ boson is produced at rest. The $Z$ decay angle $\theta_{1}$ is constrained in that case and the maximal angle $\theta_{1}^{\max }$ is given by

$$
\begin{equation*}
\sin \theta_{1}^{\max }=\frac{\left|\mathbf{p}_{\chi_{j}}^{0}\right|}{\left|\mathbf{p}_{\chi_{j}}\right|}=\frac{m_{\tilde{\ell}}}{m_{Z}} \frac{\lambda^{\frac{1}{2}}\left(m_{\chi_{j}}^{2}, m_{Z}^{2}, m_{\chi_{1}}^{2}\right)}{\left(m_{\tilde{\ell}}^{2}-m_{\chi_{j}}^{2}\right)} \leq 1 \tag{38}
\end{equation*}
$$

If $\left|\mathbf{p}_{\chi_{j}}^{0}\right|>\left|\mathbf{p}_{\chi_{j}}\right|$, the decay angle $\theta_{1}$ is not constrained and there is only the physical solution $\left|\mathbf{p}_{Z}^{+}\right|$left.

The spin basis vectors of $\tilde{\chi}_{j}^{0}$ in the $\tilde{\ell}$ rest frame are chosen by

$$
\begin{align*}
& s_{\chi_{j}}^{1}=\left(0, \frac{\mathbf{s}_{2} \times \mathbf{s}_{3}}{\left|\mathbf{s}_{2} \times \mathbf{s}_{3}\right|}\right), \quad s_{\chi_{j}}^{2}=\left(0, \frac{\mathbf{p}_{\chi_{j}} \times \mathbf{p}_{Z}}{\left|\mathbf{p}_{\chi_{j}} \times \mathbf{p}_{Z}\right|}\right) \\
& s_{\chi_{j}}^{3}=\frac{1}{m_{\chi_{j}}}\left(\left|\mathbf{p}_{\chi_{j}}\right|, E_{\chi_{j}} \hat{\mathbf{p}}_{\chi_{j}}\right) \tag{39}
\end{align*}
$$

with $\hat{\mathbf{p}}_{\chi_{j}}=\mathbf{p}_{\chi_{j}} /\left|\mathbf{p}_{\chi_{j}}\right|$. Together with $p_{\chi_{j}}^{\mu} / m_{\chi_{j}}$, the spin basis vectors form an orthonormal set.

The Lorentz invariant phase space element for the decay chain $\tilde{\ell} \rightarrow \ell \tilde{\chi}_{j}^{0} \rightarrow \ell \tilde{\chi}_{1}^{0} Z \rightarrow \ell \tilde{\chi}_{1}^{0} f \bar{f}$, in the rest frame of $\tilde{\ell}$, can be written as

$$
\begin{align*}
& \mathrm{dLips}\left(m_{\tilde{\ell}}^{2}, p_{\ell}, p_{\chi_{1}}, p_{\bar{f}}, p_{f}\right) \\
& =\frac{1}{(2 \pi)^{2}} \operatorname{dLips}\left(m_{\tilde{\ell}}^{2}, p_{\ell}, p_{\chi_{j}}\right) \mathrm{d} s_{D_{2}}  \tag{40}\\
& \quad \times \sum_{ \pm} \mathrm{d} \operatorname{Lips}\left(s_{D_{2}}, p_{\chi_{1}}, p_{Z}^{ \pm}\right) \mathrm{d} s_{D_{3}} \mathrm{dLips}\left(s_{D_{3}}, p_{\bar{f}}, p_{f}\right)
\end{align*}
$$

where $s_{D_{2}}=p_{\chi_{j}}^{2}$ and $s_{D_{3}}=p_{Z}^{2}$. The Lorentz invariant phase space elements of the sequence of two-body decays read

$$
\begin{align*}
& \operatorname{dLips}\left(m_{\tilde{\ell}}^{2}, p_{\ell}, p_{\chi_{j}}\right)=\frac{1}{8(2 \pi)^{2}}\left(1-\frac{m_{\chi_{j}}^{2}}{m_{\tilde{\ell}}^{2}}\right) \mathrm{d} \Omega  \tag{41}\\
& \mathrm{dLips}\left(s_{D_{2}}, p_{\chi_{1}}, p_{Z}^{ \pm}\right) \\
& =\frac{1}{4(2 \pi)^{2}} \frac{\left|\mathbf{p}_{Z}^{ \pm}\right|^{2}}{\left|E_{Z}^{ \pm}\right| \mathbf{p}_{\chi_{j}}\left|\cos \theta_{1}-E_{\chi_{j}}\right| \mathbf{p}_{Z}^{ \pm}| |} \mathrm{d} \Omega_{1}  \tag{42}\\
& \operatorname{dLips}\left(s_{D_{3}}, p_{\bar{f}}, p_{f}\right)=\frac{1}{8(2 \pi)^{2}} \frac{m_{Z}^{2}}{\left(E_{Z}^{ \pm}-\left|\mathbf{p}_{Z}^{ \pm}\right| \cos \theta_{2}\right)^{2}} \mathrm{~d} \Omega_{2} \tag{43}
\end{align*}
$$

where $\mathrm{d} \Omega_{i}=\sin {\underset{\sim}{\ell}}_{i} \mathrm{~d} \theta_{i} \mathrm{~d} \phi_{i}$.
The partial $\tilde{\ell}$ decay width for the decay chain (5) is given by

$$
\begin{align*}
& \Gamma\left(\tilde{\ell} \rightarrow \ell \tilde{\chi}_{1}^{0} f \bar{f}\right) \\
& =\frac{1}{2 m_{\tilde{\ell}}} \int|\mathcal{M}|^{2} \mathrm{dLips}\left(m_{\tilde{\ell}}^{2}, p_{\ell}, p_{\chi_{1}}, p_{\bar{f}}, p_{f}\right) \tag{44}
\end{align*}
$$

We use the narrow width approximation for the propagators $\Delta\left(\tilde{\chi}_{j}^{0}\right)$ and $\Delta(Z)$ :

$$
\int\left|\Delta\left(\tilde{\chi}_{j}^{0}\right)\right|^{2} \mathrm{~d} s_{D_{2}}=\frac{\pi}{m_{\chi_{j}} \Gamma_{\chi_{j}}}, \int|\Delta(Z)|^{2} \mathrm{~d} s_{D_{3}}=\frac{\pi}{m_{Z} \Gamma_{Z}}
$$

The approximation for the neutralino propagator is justified for $\left(\frac{\Gamma_{\chi_{j}}}{m_{\chi_{j}}}\right)^{2} \ll 1$, which holds in our case with $\Gamma_{\chi_{j}} \lesssim \mathcal{O}(\mathrm{GeV})$.

From (4) and (34) we obtain for the asymmetry

$$
\begin{equation*}
\mathcal{A}_{T}^{\ell, q}= \tag{45}
\end{equation*}
$$

$$
\frac{\int\left|\Delta\left(\tilde{\chi}_{j}^{0}\right)\right|^{2}|\Delta(Z)|^{2} \operatorname{sgn}\left[O_{\mathrm{odd}}^{\ell, q}\right] \sum_{D_{1}}^{a} \Sigma_{D_{2} \rho \sigma}^{a} D_{3}^{\rho \sigma} \mathrm{dLips}}{\int\left|\Delta\left(\tilde{\chi}_{j}^{0}\right)\right|^{2}|\Delta(Z)|^{2} D_{1} D_{2 \rho \sigma} D_{3}^{\rho \sigma} \mathrm{dLips}},
$$

where in the derivation of this expression we have used $\int\left|\Delta\left(\tilde{\chi}_{j}^{0}\right)\right|^{2}|\Delta(Z)|^{2} \operatorname{sgn}\left[O_{\text {odd }}^{\ell, q}\right] D_{1} D_{2 \rho \sigma} D_{3}^{\rho \sigma} \mathrm{dLips}=0$ in the numerator and $\int\left|\Delta\left(\tilde{\chi}_{j}^{0}\right)\right|^{2}|\Delta(Z)|^{2} \Sigma_{D_{1}}^{a} \Sigma_{D_{2} \rho \sigma}^{a} D_{3}^{\rho \sigma}$ $\times \mathrm{dLips}=0$ in the denominator. As can be seen from (45), the asymmetry $\mathcal{A}_{T}^{\ell, q}$ is proportional to the spin correlation terms $\Sigma_{D_{1}}^{a} \sum_{D_{2} \rho \sigma}^{a} D_{3}^{\rho \sigma}$. In the spin correlations only the term which contains the $T$-odd correlation $O_{\text {odd }}^{\ell, q}$, (2) and (3), contributes to $\mathcal{A}_{T}^{\ell, q}$.

The $T$-odd correlation $O_{\text {odd }}^{\ell, q}$ is contained in the product of the first term of (32) and the last term of (33), which leads to

$$
\begin{align*}
& \Sigma_{D_{2} \rho \sigma}^{a} D_{3}^{\rho \sigma} \supset-32 m_{\chi_{1}} \operatorname{Im}\left(O_{1 j}^{\prime \prime L} O_{1 j}^{\prime \prime R^{*}}\right)\left(R_{f}^{2}-L_{f}^{2}\right) \\
& \quad \times \varepsilon^{\rho \sigma \mu \nu} p_{\chi_{j} \rho} s_{\chi_{j} \sigma}^{a} p_{f \mu} p_{\bar{f} \nu} \tag{46}
\end{align*}
$$

In the rest frame of $\tilde{\ell},\left(p_{\ell} \cdot s_{\chi_{j}}^{a}\right)=0$ for $a=1,2$; hence, $\Sigma_{D_{1}}^{1,2}=0$ in (30) and only $\Sigma_{D_{2} \rho \sigma}^{3}$, defined in (32), contributes to the spin correlation terms in the total amplitude squared. Using the explicit representation of the fermion momentum vector, (35), and the neutralino spin vector, (39), the term with the $\varepsilon$-tensor in (46) can be written as

$$
\begin{align*}
& \varepsilon^{\rho \sigma \mu \nu} p_{\chi_{j} \rho} s_{\chi_{j} \sigma}^{3} p_{f \mu} p_{\bar{f} \nu} \\
& =-m_{\chi_{j}} \hat{\mathbf{p}}_{\ell} \cdot\left(\mathbf{p}_{f} \times \mathbf{p}_{\bar{f}}\right) \\
& =-m_{\chi_{j}}\left|\mathbf{p}_{Z}\right|\left|\mathbf{p}_{\bar{f}}\right| \sin \theta_{1} \sin \theta_{2} \sin \phi_{2} \tag{47}
\end{align*}
$$

where $\hat{\mathbf{p}}_{\ell}=\mathbf{p}_{\ell} /\left|\mathbf{p}_{\ell}\right|$. Since $0 \leq \theta_{1}, \theta_{2} \leq \pi$ the sign of the correlation $\mathbf{p}_{\ell} \cdot\left(\mathbf{p}_{f} \times \mathbf{p}_{\bar{f}}\right)$ is given by the $\operatorname{sign}$ of $\sin \phi_{2}$. Thus $O_{\text {odd }}>0$ corresponds to an integration $\int_{0}^{\pi} \mathrm{d} \phi_{2}$, while $O_{\text {odd }}<0$ corresponds to an integration $\int_{\pi}^{2 \pi} \mathrm{~d} \phi_{2}$. We therefore integrate in (44) over the entire phase space except over the angle $\phi_{2}$. The $T$-odd asymmetry can then be written as

$$
\begin{equation*}
\mathcal{A}_{T}^{f}=\frac{\left[\int_{0}^{\pi} \frac{\mathrm{d} \Gamma}{\mathrm{~d} \phi_{2}}-\int_{\pi}^{2 \pi} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} \phi_{2}}\right] \mathrm{d} \phi_{2}}{\left[\int_{0}^{\pi} \frac{\mathrm{d} \Gamma}{\mathrm{~d} \phi_{2}}+\int_{\pi}^{2 \pi} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} \phi_{2}}\right] \mathrm{d} \phi_{2}} \tag{48}
\end{equation*}
$$

The dependence of $\mathcal{A}_{T}^{f}$ on the $\tilde{\ell}_{k}-\ell-\tilde{\chi}_{j}^{0}$ couplings $a_{k j}^{\tilde{\ell}}, b_{k j}^{\tilde{\ell}}$, on the $Z-\bar{f}-f$ couplings $L_{f}, R_{f}$ and on the $Z-\tilde{\chi}_{1}^{0}-\tilde{\chi}_{j}^{0}$ couplings $O_{1 j}^{\prime \prime L, R}$ is given by

$$
\begin{equation*}
\mathcal{A}_{T}^{f} \propto \frac{\left|a_{k j}^{\tilde{\ell}}\right|^{2}-\left|b_{k j}^{\tilde{\ell}}\right|^{2}}{\left|a_{k j}^{\tilde{\ell}}\right|^{2}+\left|b_{k j}^{\tilde{\ell}}\right|^{2}} \frac{L_{f}^{2}-R_{f}^{2}}{L_{f}^{2}+R_{f}^{2}} \operatorname{Im}\left(O_{1 j}^{\prime \prime L} O_{1 j}^{\prime \prime R^{*}}\right) \tag{49}
\end{equation*}
$$

which follows from (45) and (46). Due to the first factor $\frac{\left|a_{k j}^{\tilde{E}}\right|^{2}-\left|b_{k j}^{\tilde{E}}\right|^{2}}{\left|a_{k j}^{\tilde{E}}\right|^{2}+\left|b_{k j}^{\tilde{E}}\right|^{2}}$, the asymmetry $\mathcal{A}_{T}^{f}$ will be strongly suppressed for the case $\left|a_{k j}^{\tilde{\ell}}\right| \approx\left|b_{k j}^{\tilde{\ell}}\right|$ and maximally enhanced in the
case of vanishing mixing in the slepton sector $\frac{\left|a_{k j}^{\tilde{E}}\right|^{2}-\left|b_{k j}^{\tilde{E}}\right|^{2}}{\left|a_{k j}^{\tilde{E}}\right|^{2}+\left|b_{k j}^{\tilde{E}}\right|^{2}} \approx$ $\pm 1$. Due to the second factor, $\frac{L_{f}^{2}-R_{f}^{2}}{L_{f}^{2}+R_{f}^{2}}, \mathcal{A}_{T}^{b(c)}$ is larger than $\mathcal{A}_{T}^{\ell}$, with

$$
\begin{equation*}
\mathcal{A}_{T}^{b(c)}=\frac{L_{\ell}^{2}+R_{\ell}^{2}}{L_{\ell}^{2}-R_{\ell}^{2}} \frac{L_{b(c)}^{2}-R_{b(c)}^{2}}{L_{b(c)}^{2}+R_{b(c)}^{2}} \mathcal{A}_{T}^{\ell} \simeq 6.3(4.5) \times \mathcal{A}_{T}^{\ell} \tag{50}
\end{equation*}
$$

Note that the RHS of (46), and therefore the asymmetry in (4), vanishes for $m_{\chi_{1}} \rightarrow 0$, which is related to the fact that it is possible to redefine the Weyl spinor $\chi_{1} \rightarrow e^{\mathrm{i} \alpha} \chi_{1}$ in this limit.

## 4 Numerical results

We present numerical results for the $T$-odd asymmetry $\mathcal{A}_{T}^{\ell}$ defined in (4). The values for $\mathcal{A}_{T}^{b, c}$ may be obtained from (50). We analyze numerically the decay chain $\tilde{\tau} \rightarrow$ $\tau \tilde{\chi}_{j}^{0} \rightarrow \tau \tilde{\chi}_{1}^{0} Z \rightarrow \tau \tilde{\chi}_{1}^{0} \ell \bar{\ell}, \ell=e, \mu, \tau$ where $\tilde{\chi}_{1}^{0}$ is the lightest supersymmetric particle (LSP). We assume that $\tilde{\tau}_{1}$ is the lightest sfermion and that the decays into a real $\tilde{\chi}_{j}^{0}, j=2,3$, and a real $Z$ are kinematically possible. In the numerical study below we will treat the two cases $\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{2}^{0}$ and $\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{3}^{0}$ separately. The asymmetry $\mathcal{A}_{T}^{\ell, q}$ in (4) could in principle also be studied in $\tilde{\chi}_{j}^{0}$ three-body decays if the two-body decays are kinematically forbidden [13], which will be treated elsewhere [28].

The relevant MSSM parameters are $|\mu|, \varphi_{\mu},\left|M_{1}\right|, \varphi_{M_{1}}$, $M_{2}, \tan \beta,\left|A_{\tau}\right|, \varphi_{A_{\tau}}, m_{\tilde{\tau}_{1}}, m_{\tilde{\tau}_{2}}$, and the Higgs mass parameter $m_{A}$. For all scenarios we keep $\tan \beta=10,\left|A_{\tau}\right|=$ $1000 \mathrm{GeV}, \varphi_{A_{\tau}}=0, m_{\tilde{\tau}_{1}}=300 \mathrm{GeV}, m_{\tilde{\tau}_{2}}=800 \mathrm{GeV}$ and use the GUT relation $\left|M_{1}\right|=5 / 3 \tan ^{2} \Theta_{\mathrm{W}} M_{2}$ in order to reduce the number of free parameters. We have checked that our results do not depend sensitively on this choice. We choose $m_{A}=800 \mathrm{GeV}$ to rule out decays of the neutralino into charginos and the charged Higgs bosons $\tilde{\chi}_{j}^{0} \nrightarrow \tilde{\chi}_{i}^{ \pm} H^{\mp}, i=1,2$, as well as decays into the heavy neutral Higgs bosons $\tilde{\chi}_{j}^{0} \nrightarrow \tilde{\chi}_{i}^{0} H_{2,3}^{0}$. For the calculation of the branching ratios $\operatorname{BR}\left(\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{j}^{0}\right)$ and $\operatorname{BR}\left(\tilde{\chi}_{j}^{0} \rightarrow Z \tilde{\chi}_{1}^{0}\right)$, we take into account also the decays $\tilde{\tau}_{1} \rightarrow \tilde{\chi}_{j}^{-} \nu_{\tau}, \tilde{\chi}_{j}^{0} \rightarrow$ $H_{1}^{0} \tilde{\chi}_{1}^{0}, W^{ \pm} \tilde{\chi}_{1}^{\mp}$ in addition to $\tilde{\tau}_{1} \rightarrow \tilde{\chi}_{j}^{0} \tau, \tilde{\chi}_{j}^{0} \rightarrow Z \tilde{\chi}_{1}^{0} . H_{1}^{0}$ is the lightest neutral Higgs boson, which in general is not a $C P$-eigenstate [29-31]. The decay $\tilde{\tau}_{1} \rightarrow \tau \ell \tilde{\ell}, \ell=e, \mu$ is kinematically forbidden due to our assumption that $\tilde{\tau}_{1}$ is the lightest sfermion. Other decay chains leading to the same final state are less important and will be neglected.

### 4.1 Decay chain via $\tilde{\chi}_{2}^{0}$

First we consider $\mathcal{A}_{T}^{\ell}$ for the decay chain $\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{2}^{0} \rightarrow$ $\tau Z \tilde{\chi}_{1}^{0} \rightarrow \tau \tilde{\chi}_{1}^{0} \ell \bar{\ell}$, for $\ell=e, \mu, \tau$. In Fig. 2 a we show the contour lines for the branching ratio $\operatorname{BR}\left(\tau_{1} \rightarrow \tau \tilde{\chi}_{1}^{0} \ell \bar{\ell}\right)=$ $\mathrm{BR}\left(\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{2}^{0}\right) \times \mathrm{BR}\left(\tilde{\chi}_{2}^{0} \rightarrow Z \tilde{\chi}_{1}^{0}\right) \times \mathrm{BR}(Z \rightarrow \ell \bar{\ell})$ in the $M_{2}-|\mu|$ plane for $\varphi_{M_{1}}=\pi / 2$ and $\varphi_{\mu}=0$. Small values of the phase of $\mu$ are suggested by constraints on the


Fig. 2a,b. Contour lines of the branching ratio for $\tilde{\tau}_{1} \rightarrow \tilde{\chi}_{1}^{0} \tau \ell \bar{\ell}$ and asymmetry $A_{T}^{\ell}$ defined in (4) in the $M_{2}-\mu$ plane for $\varphi_{M_{1}}=\pi / 2$ and $\varphi_{\mu}=0$, taking $\tan \beta=10, A_{\tau}=1000 \mathrm{GeV}, m_{\tilde{\tau}_{1}}=300 \mathrm{GeV}, m_{\tilde{\tau}_{2}}=800 \mathrm{GeV}$ for $M_{\tilde{E}}>M_{\tilde{L}}$. The grey areas are kinematically forbidden since here $m_{\tilde{\tau}_{1}}<m_{\chi_{2}^{0}}+m_{\tau}$ (light grey) or $m_{\chi_{2}^{0}}<m_{\chi_{1}^{0}}+m_{Z}$ (dark grey)

EDMs for a typical SUSY scale of the order of a few 100 GeV [12]. For $\operatorname{BR}\left(\tau_{1} \rightarrow \tau \tilde{\chi}_{1}^{0} \ell \bar{\ell}\right)$ we always sum over the lepton-anti-lepton pairs which couple to the $Z$. The grey areas in Fig. 2 are kinematically forbidden since here $m_{\tilde{\tau}_{1}}<m_{\chi_{2}^{0}}+m_{\tau}$ (light grey) or $m_{\chi_{2}^{0}}<m_{\chi_{1}^{0}}+m_{Z}$ (dark grey). We choose $M_{\tilde{E}}>M_{\tilde{L}}$ since in this case the $\tilde{\tau}_{1}-\tau-\tilde{\chi}_{2}^{0}$ coupling $\left|a_{12}^{\tilde{\tau}}\right|$ is larger, which implies a larger branching ratio $\mathrm{BR}\left(\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{2}^{0}\right)$ than for $M_{\tilde{E}}<M_{\tilde{L}}$. (We use the usual notation $M_{\tilde{E}} \equiv M_{R \tilde{\tau}}, M_{\tilde{L}} \equiv M_{L \tilde{\tau}}$; see (14) and (15).) $M_{\tilde{E}}>M_{\tilde{L}}$ is suggested in some scenarios with non-universal scalar mass parameters at the GUT scale [32]. Furthermore, in (14) and (15) one could have $M_{\tilde{\tau}_{L L}}<M_{\tilde{\tau}_{R R}}$ in extended models with additional D-terms [33]. In a large region of the parameter space we have $\operatorname{BR}\left(\tilde{\chi}_{2}^{0} \rightarrow Z \tilde{\chi}_{1}^{0}\right)=1$, and we take $\operatorname{BR}(Z \rightarrow \ell \bar{\ell})=0.1$. The corresponding asymmetry $\mathcal{A}_{T}^{\ell}$ is shown in Fig. 2b. The dependence of $\mathcal{A}_{T}^{\ell}$ on $M_{2}$ and $|\mu|$ is dominantly determined by $\operatorname{Im}\left(O_{12}^{\prime \prime L} O_{12}^{\prime \prime R^{*}}\right)$.

In Fig. 3 we show the $\varphi_{M_{1}}$ and $\varphi_{\mu}$ dependence of $\operatorname{BR}\left(\tilde{\tau}_{1} \rightarrow \tau \chi_{1}^{0} \ell \bar{\ell}\right)$ and of $\mathcal{A}_{T}^{\ell}$ in the full range of the phases for $|\mu|=300 \mathrm{GeV}$ and $M_{2}=280 \mathrm{GeV}$. We display in Table 1 the masses of $\tilde{\chi}_{i}^{0}, i=1, \ldots, 4$ and the total widths $\Gamma_{\tilde{\chi}_{2}}, \Gamma_{\tilde{\tau}_{1}}$ for various values of the phases. The value of $\mathcal{A}_{T}^{\ell}$ depends stronger on $\varphi_{M_{1}}$, which also determines the sign of $\mathcal{A}_{T}^{\ell}$, than on $\varphi_{\mu}$.

Based on our results on the asymmetry $\mathcal{A}_{T}^{\ell}$ in $\tilde{\tau}_{1} \rightarrow$ $\tau \tilde{\chi}_{2}^{0} \rightarrow \tilde{\chi}_{1}^{0} \tau \ell^{+} \ell^{-}$and the branching ratio we give a theoretical estimate of the number of produced $\tilde{\tau}_{1}$ 's necessary to observe the $T$-odd asymmetry in (4). The relevant quantity to decide whether $\mathcal{A}_{T}^{\ell}$ is observable (at $1 \sigma$ ), is given by $\left(\left(\mathcal{A}_{T}^{\ell}\right)^{2} \times \mathrm{BR}\right)^{-1}$, where BR stands for the branching ratio of the decay considered. The necessary number of produced

Table 1. Masses of $\tilde{\chi}_{i}^{0}, i=1, \ldots, 4$ and widths $\Gamma_{\tilde{\chi}_{2}}, \Gamma_{\tilde{\tau}_{1}}$ for various phase combinations of $\varphi_{\mu}$ and $\varphi_{M_{1}}$, taking $|\mu|=300 \mathrm{GeV}$ and $M_{2}=280 \mathrm{GeV}, \tan \beta=10, A_{\tau}=1000 \mathrm{GeV}, m_{\tilde{\tau}_{1}}=$ $300 \mathrm{GeV}, m_{\tilde{\tau}_{2}}=800 \mathrm{GeV}$ for $M_{\tilde{E}}>M_{\tilde{L}}$

| $\varphi_{\mu}$ | $\varphi_{M_{1}}$ | $m_{\tilde{\chi}_{1}}, m_{\tilde{\chi}_{2}}, m_{\tilde{\chi}_{3}}, m_{\tilde{\chi}_{4}}[\mathrm{GeV}]$ | $\Gamma_{\tilde{\chi}_{2}}[\mathrm{MeV}]$ | $\Gamma_{\tilde{\tau}_{1}}[\mathrm{MeV}]$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 135, | 234, | 306, | 358 | 4.06 |
| 0 | $\frac{\pi}{2}$ | 137, | 233, | 308, | 357 | 1.79 |
| 0 | $\pi$ | 138, | 231, | 309, | 356 | 0.09 |
| $\frac{\pi}{2}$ | 0 | 137, | 239, | 307, | 353 | 5.43 |
| $\frac{\pi}{2}$ | $\frac{\pi}{2}$ | 138, | 238, | 309, | 352 | 2.89 |
| $\frac{\pi}{2}$ | $\pi$ | 137, | 237, | 311, | 351 | 1.49 |
| $\pi$ | 0 | 138, | 245, | 309, | 347 | 7.25 |
| $\pi$ | $\frac{\pi}{2}$ | 137, | 244, | 311, | 346 | 5.78 |
| $\pi$ | $\pi$ | 136, | 243, | 313, | 345 | 4.32 |

$\tilde{\tau}_{1}$ 's should then be $\gtrsim\left(\left(\mathcal{A}_{T}^{\ell}\right)^{2} \times \mathrm{BR}\right)^{-1}$. As an example we take the point denoted by $\bullet$ in Fig. 3, with $\varphi_{\mu}=\pi / 2$ and $\varphi_{M_{1}}=\pi / 2$. For this point $\mathrm{BR} \approx 2.5 \times 10^{-2}$ and $\left|\mathcal{A}_{T}^{\ell}\right| \approx$ $3 \times 10^{-2}$ which implies that $\left(\left(\mathcal{A}_{T}^{\ell}\right)^{2} \times \mathrm{BR}\right)^{-1} \approx 4.4 \times 10^{5}$. For the decay $\tilde{\tau}_{1} \rightarrow b \bar{b} \tilde{\chi}_{1}^{0} \tau$, on the other hand, $\mathrm{BR} \approx 3.6 \times 10^{-2}$ and $\left|\mathcal{A}_{T}^{b}\right| \approx 1.9 \times 10^{-1}$, so that $\left(\left(\mathcal{A}_{T}^{b}\right)^{2} \times \mathrm{BR}\right)^{-1} \approx 7.7 \times 10^{2}$. For comparison we consider a further example with smaller $C P$-violating phases $\varphi_{\mu}=0$ and $\varphi_{M_{1}}=-0.3 \pi$ (denoted by $\otimes$ in Fig. 3). Also in this case we obtain almost the same results for $\left(\left(\mathcal{A}_{T}^{\ell, b}\right)^{2} \times \mathrm{BR}\right)^{-1}$. In these two examples, the asymmetry $\mathcal{A}_{T}^{\ell, q}$ should be measurable at an $e^{+} e^{-}$linear collider with $\sqrt{s}=800 \mathrm{GeV}$ and an integrated luminosity of $500 \mathrm{fb}^{-1}$ for $m_{\tilde{\tau}_{1}}=300 \mathrm{GeV}$. It is clear that detailed Monte Carlo studies taking into account background and

$\varphi_{M_{1}} / \pi \quad \mathcal{A}_{T}^{\ell}$ in $\%$

b
$\varphi_{\mu} / \pi$
Fig. 3a,b. Contour lines of the branching ratio for $\tilde{\tau}_{1} \rightarrow \tilde{\chi}_{1}^{0} \tau \ell \bar{\ell}$ and asymmetry $A_{T}^{\ell}$ defined in (4) in the $\varphi_{M_{1}}-\varphi_{\mu}$ plane for $|\mu|=300 \mathrm{GeV}$ and $M_{2}=280 \mathrm{GeV}$, taking $\tan \beta=10, A_{\tau}=1000 \mathrm{GeV}, m_{\tilde{\tau}_{1}}=300 \mathrm{GeV}, m_{\tilde{\tau}_{2}}=800 \mathrm{GeV}$ for $M_{\tilde{E}}>M_{\tilde{L}}$. The points denoted by $\bullet$ and $\otimes$, respectively, are for the theoretical estimate of the necessary number of produced $\tilde{\tau}_{1}$ 's (see text)
detector simulations are necessary to get a more precise prediction of the expected accuracy. However, this is beyond the scope of the present paper. For a Monte Carlo study on a $T$-odd observable in neutralino production and decay, see [34].

### 4.2 Decay chain via $\tilde{\chi}_{3}^{0}$

Next we discuss the decay chain $\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{3}^{0} \rightarrow \tau Z \tilde{\chi}_{1}^{0} \rightarrow$ $\tau \tilde{\chi}_{1}^{0} \ell \bar{\ell}$. The two decays $\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{2}^{0}$ and $\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{3}^{0}$ can be distinguished by measuring the $\tau$ energy in the $\tilde{\tau}_{1}$ rest frame. In Fig. 4a we show the contour lines for the branching ratio $\operatorname{BR}\left(\tau_{1} \rightarrow \tau_{-} \tilde{\chi}_{1}^{0} \ell \bar{\ell}\right)=\operatorname{BR}\left(\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{3}^{0}\right) \times \operatorname{BR}\left(\tilde{\chi}_{3}^{0} \rightarrow\right.$ $\left.Z \tilde{\chi}_{1}^{0}\right) \times \operatorname{BR}(Z \rightarrow \ell \bar{\ell})$ in the $M_{2}-|\mu|$ plane for $\varphi_{M_{1}}=\pi / 2$ and $\varphi_{\mu}=0$. The area $\mathrm{A}(\mathrm{B})$ is kinematically forbidden since $m_{\tilde{\chi}_{3}^{0}}<m_{\tilde{\chi}_{1}^{0}}+m_{Z}\left(m_{\tilde{\tau}_{1}}<m_{\tilde{\chi}_{3}^{0}}+m_{\tau}\right)$. The grey area is excluded since $m_{\chi_{1}^{ \pm}}<104 \mathrm{GeV}$. We choose $M_{\tilde{E}}<M_{\tilde{L}}$ since the $\tilde{\tau}_{1}-\tau-\tilde{\chi}_{3}^{0}$ coupling $\left|a_{13}^{\tilde{\tau}_{3}}\right|$ is larger, which implies a larger branching ratio $\operatorname{BR}\left(\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{3}^{0}\right)$ than for $M_{\tilde{E}}>M_{\tilde{L}}$. The total branching ratio is smaller than for the previous decay chain since $\operatorname{BR}\left(\tilde{\tau}_{1} \rightarrow \tau \tilde{\chi}_{3}^{0}\right)<.75(0.05)$ in the upper (lower) part of Fig. 4a.

The corresponding asymmetry $\mathcal{A}_{T}^{\ell}$ is shown in Fig. 4b. The asymmetry $\mathcal{A}_{T}^{\ell}$ vanishes on contours where either $\left|a_{13}^{\tilde{\tau}}\right|=\left|b_{13}^{\tilde{\tau}}\right|$ or $\operatorname{Im}\left(O_{13}^{\prime \prime L} O_{13}^{\prime \prime R^{*}}\right)=0$; see (49). On the one hand, along the contour line 0 in the lower part of Fig. 4 b we have $\left|a_{13}^{\tau}\right|=\left|b_{13}^{\tilde{\tau}}\right|$. On the other hand, along the contour line 0 in the upper part of Fig. 4 b we have $\operatorname{Im}\left(O_{13}^{\prime \prime L} O_{13}^{\prime \prime R^{*}}\right)=$ 0 . Furthermore, between the upper and the lower part of Fig. 4 b (area A), there is a further sign change of $\operatorname{Im}\left(O_{13}^{\prime \prime L} O_{13}^{\prime \prime R^{*}}\right)$. Concerning the first factor in (49), we remark that it increases for increasing $M_{2}$ and decreasing

Table 2. Masses of $\tilde{\chi}_{i}^{0}, i=1, \ldots, 4$ and widths $\Gamma_{\tilde{\chi}_{3}}, \Gamma_{\tilde{\tau}_{1}}$ for various phase combinations of $\varphi_{\mu}$ and $\varphi_{M_{1}}$, taking $|\mu|=150 \mathrm{GeV}$ and $M_{2}=450 \mathrm{GeV}, \tan \beta=10, A_{\tau}=1000 \mathrm{GeV}, m_{\tilde{\tau}_{1}}=$ $300 \mathrm{GeV}, m_{\tilde{\tau}_{2}}=800 \mathrm{GeV}$ for $M_{\tilde{E}}<M_{\tilde{L}}$

| $\varphi_{\mu}$ | $\varphi_{M_{1}}$ | $m_{\tilde{\chi}_{1}}, m_{\tilde{\chi}_{2}}, m_{\tilde{\chi}_{3}}, m_{\tilde{\chi}_{4}}[\mathrm{GeV}]$ | $\Gamma_{\tilde{\chi}_{3}}[\mathrm{MeV}]$ | $\Gamma_{\tilde{\tau}_{1}}[\mathrm{MeV}]$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 128, | 156, | 238, | 467 | 59.0 |
| 0 | $\frac{\pi}{2}$ | 132, | 153, | 238, | 466 | 68.2 |
| 0 | $\pi$ | 141, | 145, | 238, | 466 | 75.5 |
| $\frac{\pi}{2}$ | 0 | 131, | 158, | 237, | 466 | 41.5 |
| $\frac{\pi}{2}$ | $\frac{\pi}{2}$ | 136, | 154, | 237, | 466 | 49.4 |
| $\frac{\pi}{2}$ | $\pi$ | 142, | 145, | 240, | 465 | 73.8 |
| $\pi$ | 0 | 135, | 159, | 236, | 465 | 27.7 |
| $\pi$ | $\frac{\pi}{2}$ | 137, | 154, | 239, | 465 | 47.5 |
| $\pi$ | $\pi$ | 143, | 144, | 242, | 464 | 71.0 |

$|\mu|$. This behavior can be understood by observing that for $|\mu| / M_{2} \rightarrow 0$ the gaugino component of $\tilde{\chi}_{3}^{0}$ gets enhanced, resulting in $\left|b_{13}^{\tilde{\tau}}\right| /\left|a_{13}^{\tilde{\tau}}\right| \rightarrow 0$.

In Fig. 5 we show the dependence of $\operatorname{BR}\left(\tau_{1} \rightarrow \tau \chi_{1}^{0} \ell \bar{\ell}\right)$ and of $\mathcal{A}_{T}^{\ell}$ on the phases $\varphi_{M_{1}}$ and $\varphi_{\mu}$, fixing $|\mu|=150 \mathrm{GeV}$ and $M_{2}=450 \mathrm{GeV}$. For these parameters we display in Table 2 the masses of $\tilde{\chi}_{i}^{0}, i=1, \ldots, 4$ and the total widths $\Gamma_{\tilde{\chi}_{3}}, \Gamma_{\tilde{\tau}_{1}}$ for various phase combinations. Note that maximal CP-violating phases $\varphi_{\mu}, \varphi_{M_{1}}= \pm \pi / 2$ do not necessarily lead to the highest value of $\mathcal{A}_{T}^{\ell}$ due to the complex interplay of the phases in $\operatorname{Im}\left(O_{13}^{\prime \prime L} O_{13}^{\prime \prime R^{*}}\right)$. The value of $\mathcal{A}_{T}^{\ell}$ depends stronger on $\varphi_{M_{1}}$, which also determines the sign of $\mathcal{A}_{T}^{\ell}$, than on $\varphi_{\mu}$. Comparing Fig. 3b and Fig. 5b, one can see that both figures have in common the strong $\varphi_{M_{1}}$ dependence, where in a good approximation the sign of $\mathcal{A}_{T}^{\ell}$ is $\operatorname{sgn}\left(\mathcal{A}_{T}^{\ell}\right) \approx$


Fig. 4a,b. Contour lines of the branching ratio for $\tilde{\tau}_{1} \rightarrow \tilde{\chi}_{1}^{0} \tau \ell \bar{\ell}$ and asymmetry $A_{T}^{\ell}$ defined in (4) in the $M_{2}-\mu$ plane for $\varphi_{M_{1}}=\pi / 2$ and $\varphi_{\mu}=0, \tan \beta=10, A_{\tau}=1000 \mathrm{GeV}, m_{\tilde{\tau}_{1}}=300 \mathrm{GeV}, m_{\tilde{\tau}_{2}}=800 \mathrm{GeV}$ for $M_{\tilde{E}}<M_{\tilde{L}}$. The area A (B) is kinematically forbidden since $m_{\tilde{\chi}_{3}^{0}}<m_{\tilde{\chi}_{1}^{0}}+m_{Z}\left(m_{\tilde{\tau}_{1}}<m_{\tilde{\chi}_{3}^{0}}+m_{\tau}\right)$. The grey area is excluded since $m_{\chi_{1}^{ \pm}}<104 \mathrm{GeV}$

a

$$
\varphi_{\mu} / \pi
$$

$\varphi_{M_{1}} / \pi \quad \mathcal{A}_{T}^{\ell}$ in $\%$

b

Fig. 5a,b. Contour lines of the branching ratio for $\tilde{\tau}_{1} \rightarrow \tilde{\chi}_{1}^{0} \tau \ell \bar{\ell}$ and asymmetry $A_{T}^{\ell}$ defined in (4) in the $\varphi_{M_{1}-\varphi_{\mu}}$ plane for $|\mu|=150 \mathrm{GeV}$ and $M_{2}=450 \mathrm{GeV}$, taking $\tan \beta=10, A_{\tau}=1000 \mathrm{GeV}, m_{\tilde{\tau}_{1}}=300 \mathrm{GeV}, m_{\tilde{\tau}_{2}}=800 \mathrm{GeV}$ for $M_{\tilde{E}}<M_{\tilde{L}}$
$\operatorname{sgn}\left(\varphi_{M_{1}}\right)$ in Fig. 3b and $\operatorname{sgn}\left(\mathcal{A}_{T}^{\ell}\right) \approx-\operatorname{sgn}\left(\varphi_{M_{1}}\right)$ in Fig. 5b. This difference can be traced back to the different behavior of $\operatorname{Im}\left(O_{12}^{\prime \prime L} O_{12}^{\prime \prime R^{*}}\right)$ and $\operatorname{Im}\left(O_{13}^{\prime \prime L} O_{13}^{\prime \prime R^{*}}\right)$. Moreover, in Fig. 5b two points of level crossing appear at approximately $\varphi_{M_{1}} \approx$ $\pm 0.95 \pi, \varphi_{\mu} \approx \pm 0.7 \pi$.

## 5 Summary and conclusion

We have considered a $T$-odd correlation and the corresponding asymmetry in the sequential decay $\tilde{f} \rightarrow f^{\prime} \tilde{\chi}_{j}^{0} \rightarrow$ $f^{\prime} \tilde{\chi}_{1}^{0} Z \rightarrow f^{\prime} \tilde{\chi}_{1}^{0} f \bar{f}$. The analytical expressions have been given in the density matrix formalism. The contribution to the $T$-odd correlation is induced by possible $C P$-violating phases in the neutralino sector.

In a numerical study of the decay $\tilde{\tau}_{1} \rightarrow \tau \chi_{1}^{0} f \bar{f}$ we have shown that the $T$-odd asymmetry considered can be of the order of a few percent for leptonic final states. The number of produced $\tilde{\tau}$ 's necessary to observe $\mathcal{A}_{T}^{\ell}$ is at least of the order $10^{5}$, which may be accessible at future collider experiments. For a semi-leptonic final state like $\tilde{\chi}_{1}^{0} \tau \bar{b} b$ the $T$-odd asymmetry is larger by a factor 6.3 . If the $T$-odd asymmetry $\mathcal{A}_{T}^{b}$ of such a semi-leptonic final state could be measured with similar accuracy the number of produced $\tilde{\tau}$ 's necessary to observe $\mathcal{A}_{T}^{b}$ is of the order $10^{3}$.

Acknowledgements. We thank S. Hesselbach and W. Majerotto for useful discussions. This work is supported by the "Fonds zur Förderung der wissenschaftlichen Forschung" (FWF) of Austria, projects No. P13139-PHY and No. P16592-N02, by the European Community's Human Potential Programme under contract HPRN-CT-2000-00149 and by Acciones Integradas Hispano-Austriaca. T.K. acknowledges financial support from the European Commission Research Training Site contract HPMT-2000-00124. O.K. was supported by the Bayerische Julius-Maximilians Universität Würzburg. This work was also supported by the Deutsche Forschungsgemeinschaft (DFG) under contract Fr 1064/5-1.

## References

1. M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49, 652 (1973)
2. G.R. Farrar, M.E. Shaposhnikov, Phys. Rev. Lett. 70, 2833 (1993); Erratum 71, 210 (1993) [hep-ph/9305274]; P. Huet, E. Sather, Phys. Rev. D 51, 379 (1995) [hep-ph/9404302]; M. Carena, M. Quiros, C.E. Wagner, Phys. Lett. B 380, 81 (1996) [hep-ph/9603420]
3. For a recent review see, for example, A. Masiero, O. Vives, New J. Phys. 4, 4 (2002)
4. E.D. Commins et al., Phys. Rev. A 50, 2960 (1994)
5. P.G. Harris et al., Phys. Rev. Lett. 82, 904 (1999)
6. M.V. Romalis, W.C. Griffith, E.N. Fortson, Phys. Rev. Lett. 86, 2505 (2001); J.P. Jacobs et al., Phys. Rev. Lett. 71, 3782 (1993)
7. B.C. Regan, E.D. Commins, C.J. Schmidt, D. DeMille, Phys. Rev. Lett. 88, 071805 (2002)
8. S. Abel, S. Khalil, O. Lebedev, Nucl. Phys. B 606, 151 (2001) [hep-ph/0103320]
9. J. Ellis, S. Ferrara, D.V. Nanopoulos, Phys. Lett. B 114, 231 (1982); W. Buchmüller, D. Wyler, Phys. Lett. B 121, 321 (1983); J. Polchinski, M.B. Wise, Phys. Lett. B 125, 393 (1983); J.M. Gerard, W. Grimus, A. Masiero, D.V. Nanopoulos, A. Raychaudhuri, Nucl. Phys. B 253, 93 (1985); P. Nath, Phys. Rev. Lett. 66, 2565 (1991); Y. Kizukuri, N. Oshimo, Phys. Rev. D 45, 1806 (1992); D 46, 3025 (1992); T. Falk, K.A. Olive, Phys. Lett. B 375, 196 (1996)
10. M. Dine, A. Kagan, S. Samuel, Phys. Lett. B 243, 250 (1990); S. Dimopoulos, G.F. Giudice, Phys. Lett. B 357, 573 (1995) [hep-ph/9507282]; A. Pomarol, D. Tommasini, Nucl. Phys. B 466, 3 (1996) [hep-ph/9507462]; A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Lett. B 388, 588 (1996) [hep-ph/9607394]; J. Hisano, K. Kurosawa, Y. Nomura, Phys. Lett. B 445, 316 (1999) [hep-ph/9810411]; Nucl.

Phys. B 584, 3 (2000) [hep-ph/0002286]; J.A. Bagger, J.L. Feng, N. Polonsky, R.J. Zhang, Phys. Lett. B 473, 264 (2000) [hep-ph/9911255]; J. Bagger, J.L. Feng, N. Polonsky, Nucl. Phys. B 563, 3 (1999) [hep-ph/9905292]; K. Agashe, M. Graesser, Phys. Rev. D 59, 015007 (1999) [hepph/9801446]; J.L. Feng, C.F. Kolda, N. Polonsky, Nucl. Phys. B 546, 3 (1999) [hep-ph/9810500]
11. T. Ibrahim, P. Nath, Phys. Rev. D 57, 478 (1998); D 58, 111301 (1998); Erratum D 60, 099902 (1998); D 61, 093004 (2000); D 58, 111301 (1998); A. Bartl, T. Gajdosik, W. Porod, P. Stockinger, H. Stremnitzer, Phys. Rev. D 60, 073003 (1999); M. Brhlik, G.J. Good, G.L. Kane, Phys. Rev. D 59, 115004 (1999) [hep-ph/9810457]; A. Bartl, T. Gajdosik, E. Lunghi, A. Masiero, W. Porod, H. Stremnitzer, O. Vives, Phys. Rev. D 64, 076009 (2001) [hepph/0103324]
12. V.D. Barger, T. Falk, T. Han, J. Jiang, T. Li, T. Plehn, Phys. Rev. D 64, 056007 (2001) [hep-ph/0101106]
13. S.Y. Choi, H.S. Song, W.Y. Song, Phys. Rev. D 61, 075004 (2000) [hep-ph/9907474]
14. Y. Kizukuri, Phys. Lett. B 193, 339 (1987); Y. Kizukuri, N. Oshimo, Phys. Lett. B 220, 293 (1989); S.Y. Choi, M. Drees, B. Gaissmaier, J.S. Lee, Phys. Rev. D 64, 095009 (2001) [hep-ph/0103284]; S.Y. Choi, M. Drees, J.S. Lee, J. Song, Eur. Phys. J. C 25, 307 (2002) [hep-ph/0204200]
15. A. Bartl, T. Kernreiter, W. Porod, Phys. Lett. B 538, 59 (2002) [hep-ph/0202198]
16. N. Oshimo, Mod. Phys. Lett. A 4, 145 (1989)
17. G. Valencia, hep-ph/9411441, and references therein; G.C. Branco, L. Lavoura, J.P. Silva, CP violation (Oxford University Press, New York 1999)
18. M.B. Gavela, F. Iddir, A. Le Yaouanc, L. Oliver, O. Pene, J.C. Raynal, Phys. Rev. D 39, 1870 (1989); W. Bernreuther, O. Nachtmann, Phys. Rev. Lett 63, 2787 (1989); Phys. Lett. B 268, 424 (1991); L.M. Sehgal, M. Wanninger, Phys. Rev. D 46, 1035 (1992); Erratum D 46 (1992) 5209; P. Heiliger, L.M. Sehgal, Phys. Rev. D 47, 4920 (1993); W. Bernreuther, A. Brandenburg, Phys. Lett. B 314, 104 (1993); D. Chang, W.Y. Keung, I. Phillips, Phys. Rev. D 48, 3225 (1993) [hep-ph/9303226]; F. Cuypers, S. Rindani, Phys. Lett. B 343, 333 (1995); A. Bartl, E. Christova, W. Majerotto, Nucl. Phys. B 460, 235 (1996); Erratum B 465, 365 (1996); S. Bar-Shalom, D. Atwood, G. Eilam, R.R. Mendel, A. Soni, Phys. Rev. D 53, 1162 (1996) [hepph/9508314]; S. Bar-Shalom, D. Atwood, A. Soni, Phys. Lett. B 419, 340 (1998) [hep-ph/9707284]
19. TESLA Technical Design Report, Part: III Physics at an $e^{+} e^{-}$Linear Collider, edited by R.-D. Heuer, D. Miller, F. Richard, P. Zerwas, DESY 2001-011, hep-ph/0106315; T. Abe et al. [American Linear Collider Working Group Collaboration], Linear collider physics resource book for Snowmass 2001; Higgs and supersymmetry studies, in Proceedings of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001), edited by N. Graf, hep-ex/0106056; K. Abe et al., JLC Roadmap Report, presented at the ACFA LC Symposium, Tsukuba, Japan 2003, http://lcdev.kek.jp/RMdraft/
20. C.J. Damerell, D.J. Jackson, Prepared for 1996 DPF / DPB Summer Study on New Directions for High-Energy Physics (Snowmass 96), Snowmass, Colorado, 25 June12 July 1996; K. Abe et al. [SLD Collaboration], Phys. Rev. Lett. 88, 151801 (2002) [hep-ex/0111035]; J. Abdallah [DELPHI Collaboration], hep-ex/0311003
21. W.M. Yang, D.S. Du, Phys. Rev. D 67, 055004 (2003) [hep-ph/0211453]
22. J.G. Körner, M.C. Mauser, hep-ph/0306082
23. M.M. Nojiri, Phys. Rev. D 51, 6281 (1995); M.M. Nojiri, K. Fujii, T. Tsukamoto, Phys. Rev. D 54, 6756 (1996) [hepph/9606370]; E. Boos, H.U. Martyn, G. Moortgat-Pick, M. Sachwitz, A. Sherstnev, P.M. Zerwas, hep-ph/0303110
24. H.E. Haber, G.L. Kane, Phys. Rept. 117, 75 (1985)
25. A. Bartl, K. Hidaka, T. Kernreiter, W. Porod, Phys. Lett. B 538, 137 (2002); Phys. Rev. D 66, 115009 (2002)
26. H.E. Haber, Proceedings of the 21st SLAC Summer Institute on Particle Physics, Stanford 1993, p. 231, edited by L. DeProcel, Ch. Dunwoodie
27. G. Moortgat-Pick, H. Fraas, A. Bartl, W. Majerotto, Eur. Phys. J. C 9, 521 (1999)
28. K. Hohenwarter-Sodek, Diploma thesis, University of Vienna (2003), in German
29. A. Pilaftsis, Phys. Lett. B 435, 88 (1998) [hep-ph/9805373]
30. M. Carena, J. Ellis, A. Pilaftsis, C.E.M. Wagner, Nucl. Phys. B 586, 92 (2000)
31. S.Y. Choi, M. Drees, J.S. Lee, Phys. Lett. B 481, 57 (2000)
32. H. Baer, C. Balazs, S. Hesselbach, J.K. Mizukoshi, X. Tata, Phys. Rev. D 63, 095008 (2001) [hep-ph/0012205] (and references therein)
33. S. Hesselbach, F. Franke, H. Fraas, Eur. Phys. J. C 23, 149 (2002) [hep-ph/0107080] (and references therein)
34. S.Y. Choi, M. Drees, B. Gaissmaier, J. Song, hepph/0310284

